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False Two-Body Problem Reduction and Formulation

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Abstract— In mechanics, we are using a kind of equation that says that the two masses existing separately can be reduced into one single form such that the reduced form lies within the limit between the lower than small mass to half of the one mass for every kind of masses taken in reference. But, in this paper I am going to show such deduction of equation is not correct. In short, I can say that it is derived with correct vector geometry but in wrong concept of central force exerting phenomenon emphasizing that mass can't be reduced in such a manner.

Index terms— reduced mass equation, central forces, vector geometry, Newton's third law, equi-potential surface, Newton's law of Gravitation.

1 INTRODUCTION

In physics, there are some such deductions which are existing as abnormal in comparison to the other results of direct observations relative to others without direct observations. Specially, in Modern physics, we have such scenarios creating the division of whole physics in two parts as Classical physics and Modern physics. Newtonian physics which states that the length, mass and time are the fundamentally independent, termed as Classical but for the same, Einstein physics says the dependent, termed as Modern. The main key for such departure of physics is that Einstein physics takes the velocity of light as constant to all frame of references and it is one of his major postulate of 'Special Theory of Relativity' but not so in Newtonian physics. In the same way, there exists an equation in mechanics that explains that two body mass problems can be reduced to single body mass form that acts as if the two bodies are single. But, before deducing such relation that gives the unique concept, we must have proper observations in the deduction methods and its proper mathematical applications. In the course, I have found that reduced mass equation have correct mathematical application but wrong concept in central force exerting phenomenon resulting such unexpected prediction in the mechanics, which I am going to show below.

2 BODY

Let us consider two masses m_1 and m_2 placed at vector distances r_1 and r_2 respectively from point 'o' with separation of vector distance r between two masses as shown in figure. The concept of reduced mass equation is evaluated by taking the reference of central forces along with vector geometry as described below:

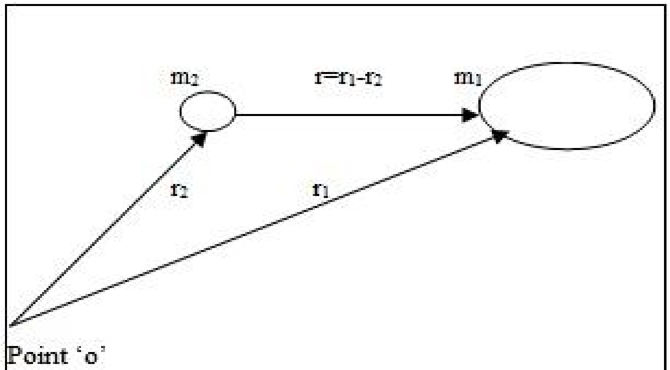

Fig. 1.showing the diagram for central forces in vector forms.

Calculated terms:
Central force exerted by the m_2 on m1 i.e. F_{12} is given by

$$F_{12} = m_1\left(\frac{d^2r_1}{dt^2}\right) \ldots.(1)$$

And similarly, central force exerted by m_1 on m_2 i.e. F_{12} is given by

$$F_{21} = m_2\left(\frac{d^2r_2}{dt^2}\right) \ldots.(2)$$

These equations are equated by taking reference according to the Newton's third law, such that F_{12}=-F_{21}.
But is it appropriate to say that the central force exerted by these masses on each other is linked by above equations? And furthermore, is it the Newton's third law that equates one another in magnitudes of force exertion? No, this kind of linkage is not the appropriate one that can give the exactness in the systematic measurement of force magnitudes, it is clarified below. If we proceed like this kind of system, we will finally attain an equation as,

$$\frac{d^2r}{dt^2} = F_{21}\left(\frac{1}{m_1}+\frac{1}{m_2}\right) \ldots.(3)$$

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Now, if we proceed the equations (1) and (2) in the way below, we will obtain an unexpected result for the equal masses taking in reference.

We have, $F_{21}=-F_{12}$

$$\text{i.e.}\ m_1\left(\frac{d^2r_1}{dt^2}\right) = -m_2\left(\frac{d^2r_2}{dt^2}\right)$$

$$\text{i.e.}\ m_1\left(\frac{d^2r_1}{dt^2}\right) + m_2\left(\frac{d^2r_2}{dt^2}\right) = 0$$

$$\text{for equal masses,}\left(\frac{d^2r_1}{dt^2}\right) + \left(\frac{d^2r_2}{dt^2}\right) = 0$$

$$\text{i.e.}\left(\frac{d^2(r_2+r_1)}{dt^2}\right) = \left(\frac{d^2(z)}{dt^2}\right)$$

Where, z is any vector which is equal to sum of two vectors r_1 and r_2. But, to have the differentiation zero, either the vector 'z' must be zero or it must be a constant vector.

$$\text{i.e.}\ (r_2 + r_1) = 0 \text{ or } z \ldots (4)$$

Thus, for $r_1+r_2=o$, this equation shows that r_2 is acting exactly opposite to the r_1, but from the figure we don't have such condition. This condition amplify drawback of reduced mass equation. According to this condition, both distance vectors must act along the same line in opposite direction with equal magnitude, the case in not in the real form what we have expected. This condition implies that the point 'o' is lying something in between the line joining two masses and both of them have equal influence of central forces acting oppositely at point 'o' for equal masses. If the masses are varied such that $m_1>m_2$ then there will be variation of the vectors' magnitudes such that $r_1<r_2$ in between the line joining the two masses to satisfy the condition for equal force magnitudes. **But if we consider the condition like as shown in figure, we need to understand that how the two masses are making their influence of central forces at the point 'o' through the vector distances r_1 by m_1 and r_2 by m_2 respectively which, is all total different then what we determine. If we take the Newton's 3rd law as a reference to equate the forces F_{21} and F_{12} in magnitude scale for such position of two masses along with distance vectors as shown above then, what is representation of vector geometry for the point 'o' lying at the line joining the two masses, to equate the force magnitudes F_{12} and F_{21} as shown in figure (3) below?** What actually I mean that if the force magnitudes are acting along a straight line or say if they are acting opposite to one another with equal magnitude of forces at the point 'o' in the figure(3) below, is same as above figure or not? Obviously not, as the vector geometry of above figure is different than the figure below. It means to equate the forces in one another in magnitude scale acting opposite to one another; there must be linear combination two forces. It will be more clear if we deal this problem taking a unit mass placed at position of point 'o', such that on keeping the unit mass there, two masses m_1 and m_2 will setup their influence of central forces to the unit mass and the unit mass will react depending upon the magnitude of two masses. If the point 'o' is point where, there is equal influence of force magnitudes exerted by the two masses oppositely, the unit mass will not show any reaction, it means an equal force is acting at the adjacent sides of the unit mass keeping the unit mass in static equilibrium. Yes, it's only the case we can take $F_{21}=-F_{12}$ in reality. But for the condition like as in above figure, if we place the unit mass in place of point 'o' then the mass will move along the vector 'z' as we have derived above in '4' and representation of vector 'z' is as shown in figure below.

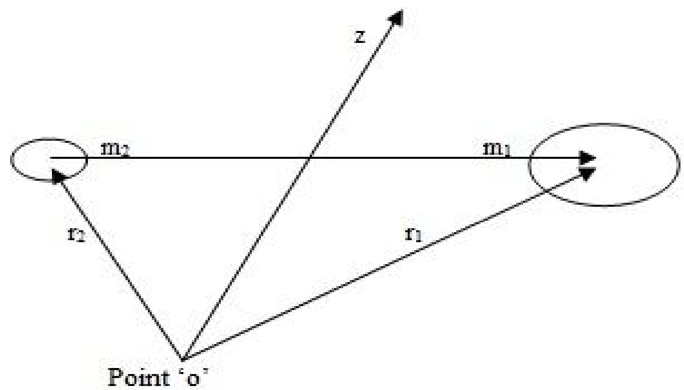

Fig .2. shows the actual direction of resultant force due to combine effect of central forces.

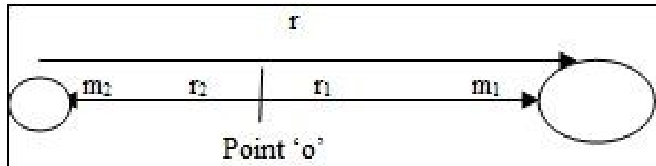

Fig.3. Showing the exertion of central forces along the vector distance r.

If we consider for equalization of central forces acting along the vector distance r between the two masses m_1 and m_2, there will be always r vector constant as two masses always apart same in magnitude r**. It is independent of change of its origin or say point 'o', it may lie at any position in a plane containing the two masses creating the variance of the vector distances r_1 and r_2. Thus, the differentiation of the vector r will always be zero as it is the constant vector**. So, the equation '3' will be

$$0 = F_{21}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\text{i.e.}\ m_1 = -m_2 \ldots (5)$$

This condition is not fair for our consideration too. But if we take any constant 'k' for the equalization of two forces F_{21} and F_{12} such that $F_{21}=kF_{12}$, we will get,

$$0 = F_{21}\left(\frac{1}{m_1} - \frac{k}{m_2}\right)$$

$$\text{i.e.}\ k = m_2/m_1 \ldots (6)$$

This shows that $F_{21}>F_{12}$ as k<1.

2.1 Illustration to be F_{12}=-F_{21}

If we take that the two masses are exerting their central forces along the vector distance r, then let us deal the phenomenon in a different way than it is described above. Such as

$$F_{21} = -F_{12}$$

$$\text{i.e.}\, m_1\left(\frac{d^2r}{dt^2}\right) = -m_2\left(\frac{d^2-r}{dt^2}\right)$$

As vector distance r on RHS is just opposite directional magnitude for LHS and vice-versa. So, the above equation in vector form will convert to the magnitude form.

$$\text{i.e.}\, m_1\frac{d^2r}{dt^2} = m_2\frac{d^2r}{dt^2}$$

$$\text{i.e.}\, m_1 = m_2$$

This clarifies that two masses varying in magnitudes will not exert an equal magnitude of central forces to their respective positions in absence of one another along the vector distance r. To be region of equal influence of central forces exerted by two masses respectively in absence of one another there must be equal mass magnitudes. It means that if we remove mass m_2 from its position and measure the central force intensity at position of m_2 caused by mass m_1, we will not get same result of central force intensity caused by mass m_1 in replacing the mass m_2. So, only the way we can equate the two such central forces is when the two masses are of equal magnitudes. Furthermore, from the figure (3) above, a unit mass placed in point 'o' will gain static equilibrium due to equal influence of central forces caused by two masses at the adjacent sides causing no motion of unit mass. At this case we can't have vector distance link up to the unit mass that could bring the unit mass in motion. But the vector distance r will be constant always with no effect of change in positions of point 'o'.

2.2 Extension to be F_{12}=-F_{21}

Another way, we can interpret to be F_{12}=-F_{21} is referring to the Equi-potential surface taking reference with the Newton law of Gravitation i.e. $\mathbf{F_1}=Gm_1m/r_1^2$ taking consideration of the force between a unit mass m with mass m_1 and $\mathbf{F_2}=Gm_2m/r_2^2$ taking consideration of the force between the same unit mass m with mass m_2 respectively at the same time. According to the equi-potential phenomenon, a unit mass placed in between two masses which are exerting their central forces to this unit mass, will react or say shows its reaction in terms of motions under the combine strength influences of such central forces such that for the regions where the influence of two central forces are of equal magnitudes, there it may or may not be exactly opposite directions. It means, if the unit mass shows no motion or some kind of motion (that may be in any direction) there it may be the region of equi-potential region. Thus, to be equi-potential surface, the motion of unit mass must be either motionless or it must move in such way without discriminating the two central forces. It will be clear more from the figure below.

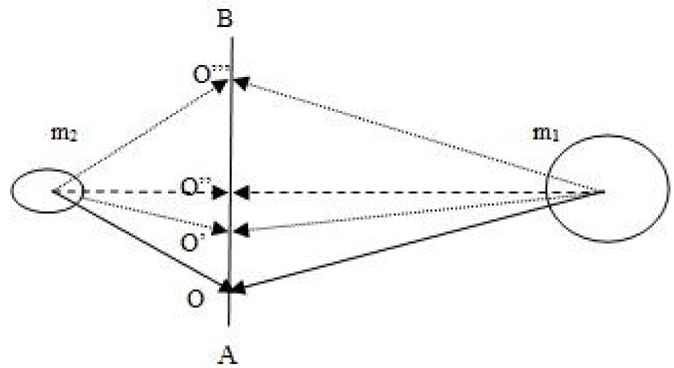

Fig.4. showing the plane of equi-potential surface (AB) between the two masses exerting the influence of central forces in unit mass kept in between them.

The points O,O',O",O"' and so on lying on the line AB for X-Y plane are only the points where the central force magnitudes exerted by two masses are equal but not in opposite except than the point O". The point O" is only such point where the central force magnitudes are equal and exactly opposite so, the unit mass will be motionless at this particular point and we can take F_{12}=-F_{21}. Other than this point, unit mass will move along the line AB such that if the position of unit mass is below than the point O", unit mass will move towards up right direction and vice- versa. But beside the points on the line AB or say equi-potential surface the two magnitudes of central forces couldn't be equal; at these points the two central forces will show their dominancy, it means on left part of the line AB, mass m_2 will show its dominancy and same way by m_1 on right part. Only the way we can link the two forces equal to one another is taking a scalar 'k' in the multiple of one two forces for another force as like in previous case. But simply it could be easily solved by using the resolution of two vectors taking the two respective central forces.

And more over, for the two masses fixing in two positions as like in above, there will be only one line or surface of equi-potential. If there is vary in mass magnitudes, the shift of such line will be replaced towards lower mass in between the line joining them. If say mass $m_2<<m_1$, such line will be very near to the centre of m_2 implying that mass m_2 will not take part for central force exertion to a unit mass placed at any points outside its surface, like keeping the unit mass m in between an iron ball(m_2) on the surface of Earth(m_1). This shows that central force application to deduce the reduced mass equation is not correct.

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3 CONCLUSION

Thus, overall we can conclude that the reduced mass equation is false in reality, it is derived with correct vector geometry but with wrong concept of central force exerting phenomenon.

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